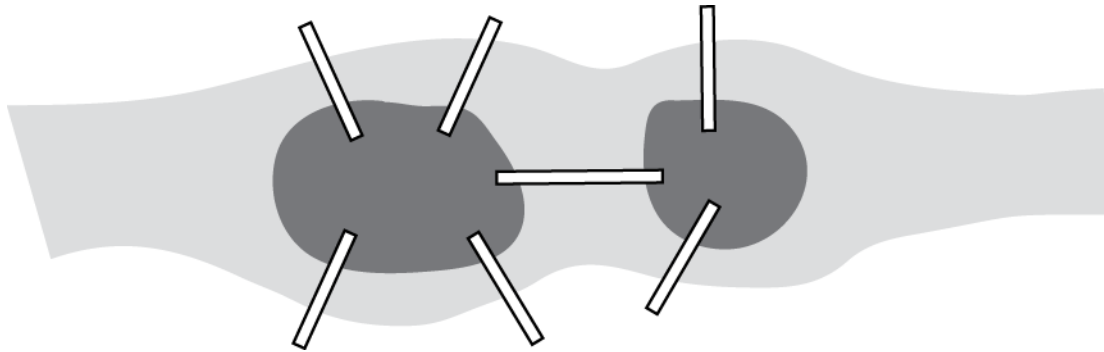


Networks and Graphs: Circuits, Paths, and Graph Structures

VII.A Student Activity Sheet 1: Euler Circuits and Paths

The Königsberg Bridge Problem

The following figure shows the rivers and bridges of Königsberg. Residents of the city occupied themselves by trying to find a walking path through the city that began and ended at the same place and crossed every bridge **exactly once**.

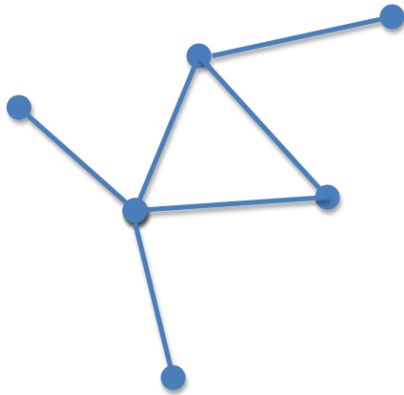


1. If you were a resident of Königsberg, where would you start your walk and what path would you choose?

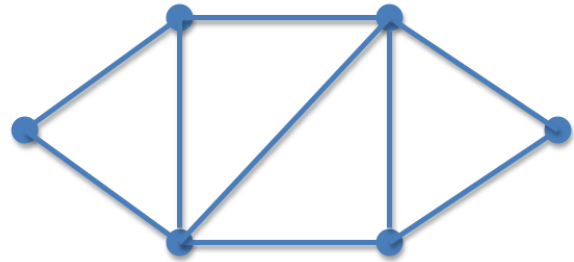
Networks and Graphs: Circuits, Paths, and Graph Structures

VII.A Student Activity Sheet 1: Euler Circuits and Paths

2. What about when you visit the Eastern and Western wildflower gardens that have fabulous sculptures in addition to beautiful flowers along the walkways. You want to see each display without backtracking (seeing something you have already seen). Where would you start your walk and what path would you choose?



Western garden

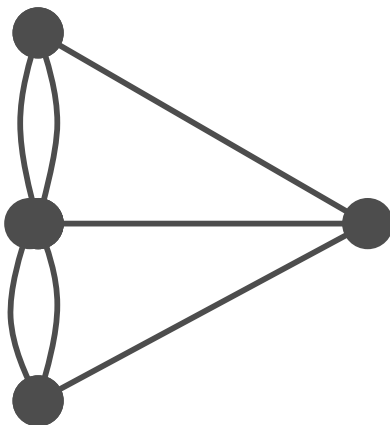


Eastern garden

Networks and Graphs: Circuits, Paths, and Graph Structures

VII.A Student Activity Sheet 1: Euler Circuits and Paths

When Leonhard Euler, a famous mathematician, turned his attention to the Bridge problem, his first step was to **model** the bridges of Königsberg with a simple graph. The points, or **vertices**, represented land and the **edges** represented the bridges connecting them. Euler's map of Königsberg, while much simpler, conveyed all the necessary information about which parts of land were connected by which bridges. It looked something like the following:

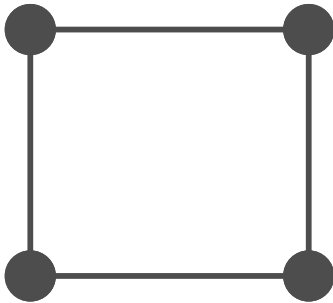


The Bridge problem is now stated: *Given a graph, find a path through the vertices (points) that uses every edge exactly once.* Such a path is called a *Euler path*. If a Euler path begins and ends at the same vertex, it is called a *Euler circuit*.

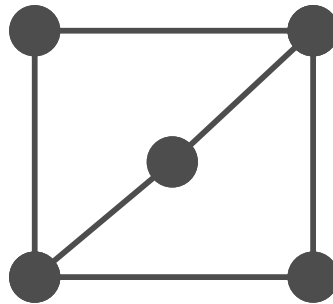
Networks and Graphs: Circuits, Paths, and Graph Structures

VII.A Student Activity Sheet 1: Euler Circuits and Paths

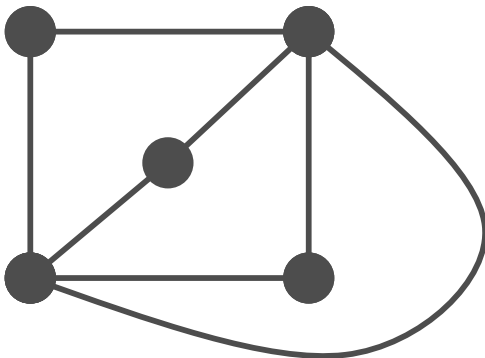
3. For the following graphs, decide which have Euler circuits and which do not.



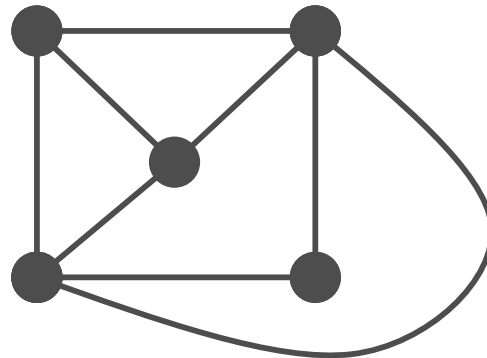
Graph I



Graph II



Graph III



Graph IV

Networks and Graphs: Circuits, Paths, and Graph Structures

VII.A Student Activity Sheet 1: Euler Circuits and Paths

4. The *degree* of a vertex is the number of edges that meet at the vertex. Determine the degree of each vertex in Graphs I-IV.

5. For the graphs from Question 3 that have Euler circuits, how many vertices have an odd degree?

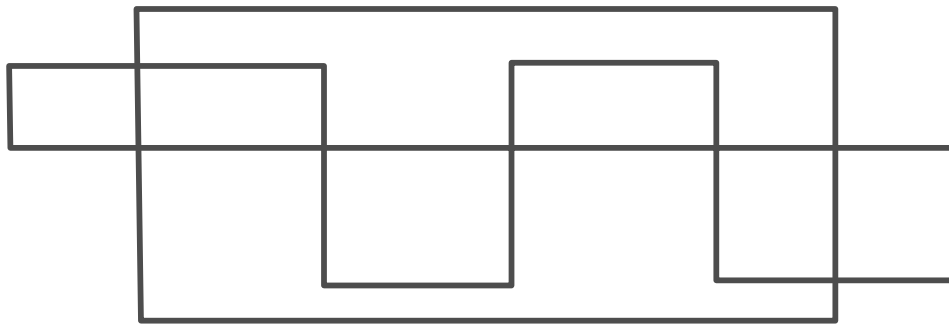
6. For the graphs from Question 3 that have Euler circuits, how many vertices have an even degree?

7. Form a **conjecture** about how you might quickly decide whether a graph has a Euler circuit, and explain why your conjecture seems reasonable.

Networks and Graphs: Circuits, Paths, and Graph Structures

VII.A Student Activity Sheet 1: Euler Circuits and Paths

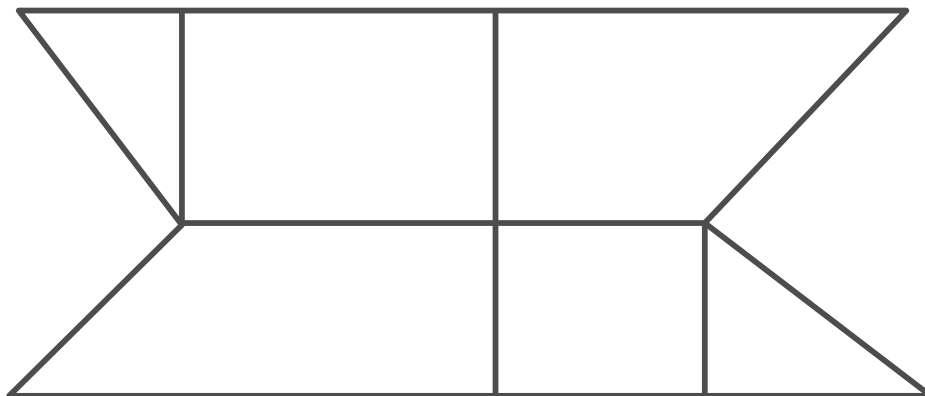
8. What does your conjecture tell you about the Königsberg Bridge problem and the garden scenario?
9. Your friend Chet calls you on his cell phone and tells you that he has discovered a large rock embedded with gems! He is somewhere in your favorite hiking area, which has many interconnected paths, as shown below. Chet does not know exactly where he is, but he needs your help to carry the rock. To find him, you decide it would be most efficient to jog along all the paths in such a way that no path is covered twice. Find this efficient route on the map below or explain why no such route exists.



Networks and Graphs: Circuits, Paths, and Graph Structures

VII.A Student Activity Sheet 1: Euler Circuits and Paths

10. You have been hired to paint the yellow median stripe on the roads of a small town. Since you are being paid by the job and not by the hour, you want to find a path through the town that traverses each road only once. In the map of the town's roads below, find such a path or explain why no such path exists.



11. **REFLECTION:** For what situation(s) is it satisfactory to have only a *path* exist and not a *circuit*?

Networks and Graphs: Circuits, Paths, and Graph Structures

VII.A Student Activity Sheet 1: Euler Circuits and Paths

- 12. EXTENSION:** Determine some other real-world problems whose solutions may involve finding Euler circuits or paths in graphs. There are a variety of road-traversing problems: delivering mail, garbage/recyclable collecting in a city, sweeping/cleaning streets, and so on.

For each situation, describe what real-world complications exist that might make the problem more difficult. For example, when delivering mail, most streets have houses on either side of the street and the postal worker may decide to go up one side of the street and down the other. If the city has alleyways, perhaps the garbage collectors just need to travel down the alleys.

Be prepared to make a short presentation of your findings to the class.

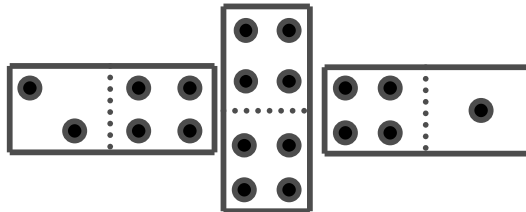
Networks and Graphs: Circuits, Paths, and Graph Structures

VII.A Student Activity Sheet 2: Dominoes

Dominoes are rectangular tiles divided into two squares. Each square has a number (usually represented by a series of dots) from 0 to 6. A double-six set of dominoes has tiles of every possible combination of these numbers, from 0-0 to 6-6. Each possible combination of numbers appears only once in a set, so a complete set of dominoes contains 28 tiles. Two tiles from a double-six set of dominoes are shown below:



In many games, you must place the dominoes next to each other in such a way that squares with identical numbers are placed next to each other. Doubles are traditionally rotated before they are placed. Adhering to these rules, the following shows a legal placement of three dominoes:



Domino Placement Problem

Can all the dominoes in a double-six set be placed in a single line of tiles adhering to the placement rules previously described? Can they all be placed so that the single line loops back to the first domino? The remainder of this activity sheet will help you answer these questions.

1. Decide how to use a graph to model the Domino Placement problem. Carefully define what your vertices represent and how you know when two vertices are connected by an edge.

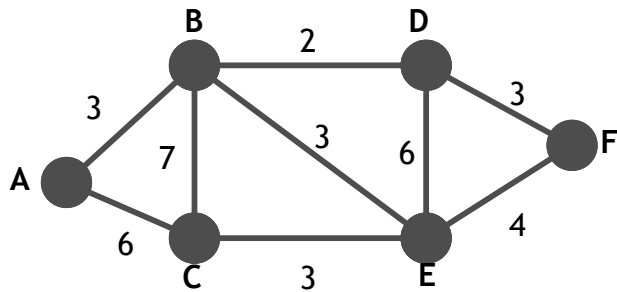
Networks and Graphs: Circuits, Paths, and Graph Structures

VII.A Student Activity Sheet 3: Weighted Graphs

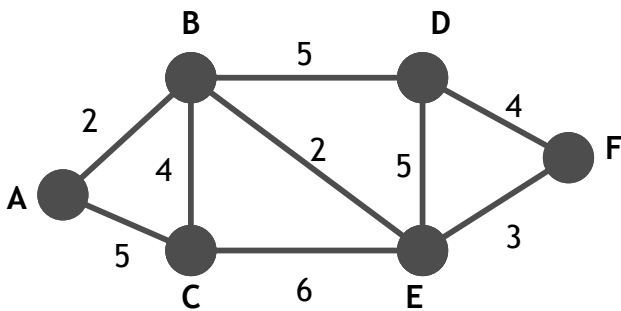
The Snowplow Problem

As the new snowplow operator, you must decide the best route through three cities. In each city, you need to plow all the roads and return to your starting place, but you must also keep from backtracking as much as possible.

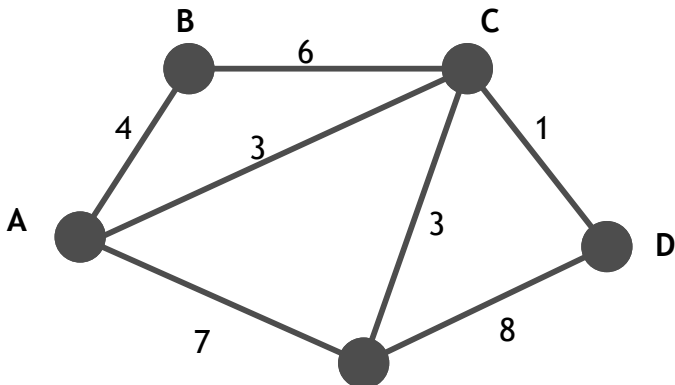
- Construct two snowplow routes through each of the following cities and indicate the time it will take to travel each route. The time it takes to traverse each road (in hours) is indicated in the graph.



City I



City II



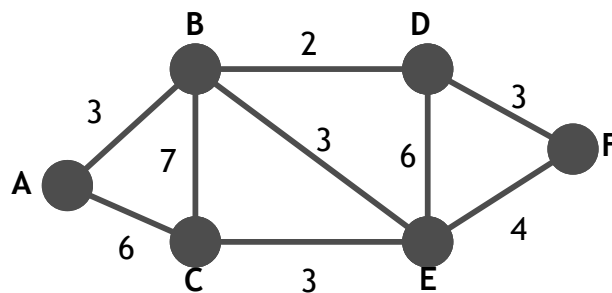
City III

Networks and Graphs: Circuits, Paths, and Graph Structures

VII.A Student Activity Sheet 3: Weighted Graphs

2. **REFLECTION:** How would you solve the Snowplow problem for a graph that has no vertices of an odd degree?

3. City I has two vertices of an odd degree.



a. Find these vertices and the shortest path between them.

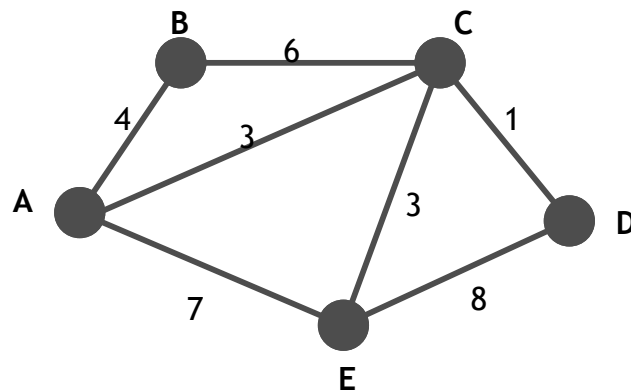
b. For each edge in this shortest path, put in a second copy of the edge.

c. At this point, your graph should have no vertices of an odd degree. Find a Euler circuit and compare this path with the paths you found for City I.

Networks and Graphs: Circuits, Paths, and Graph Structures

VII.A Student Activity Sheet 3: Weighted Graphs

4. Follow the procedure outlined in Question 3 to find a solution to the Snowplow problem for City III.



Networks and Graphs: Circuits, Paths, and Graph Structures

VII.A Student Activity Sheet 3: Weighted Graphs

5. **EXTENSION:** Prior to the beginning of school, a huge task occurs at almost every school in the nation—cleaning the floors! Whether it be waxing, steam cleaning, or mopping, it is critical that the floors be ready for the first day of school. Your task is to design the plan for this cleaning project.

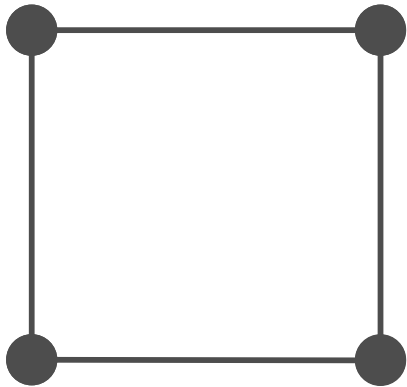
- Draw a graph of the hallways in your school or portion of your school (at least six edges).
- Assign weights to each edge according to the width and length of the corresponding hallway and how long it will take to complete that edge or hallway.

Design a path through your school or portion of your school and determine the total number of hours required to complete the cleaning of the floors. Be prepared to share your design with the class.

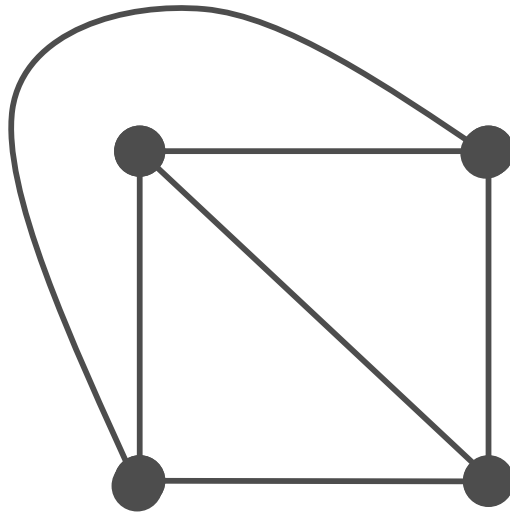
Networks and Graphs: Circuits, Paths, and Graph Structures

VII.A Student Activity Sheet 4: Hamiltonian Circuits and Paths

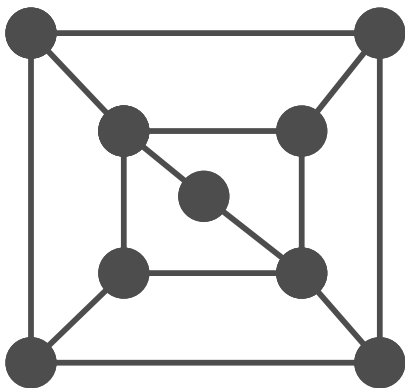
2. A path on a graph that goes through each vertex once is called a *Hamiltonian path*. A path that starts and stops at the same vertex and goes through each vertex once is called a *Hamiltonian circuit*. Which of the following graphs have a Hamiltonian circuit?



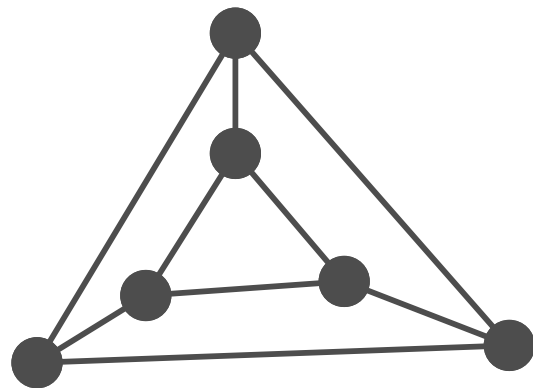
Graph I



Graph II



Graph III



Graph IV

Student: _____ Class: _____ Date: _____

Networks and Graphs: Circuits, Paths, and Graph Structures

VII.A Student Activity Sheet 4: Hamiltonian Circuits and Paths

3. Form a conjecture about when you think a graph might have a Hamiltonian circuit.

4. Share your conjecture with others and try to find examples of graphs that disprove your conjecture. These are called *counterexamples*.

Networks and Graphs: Circuits, Paths, and Graph Structures

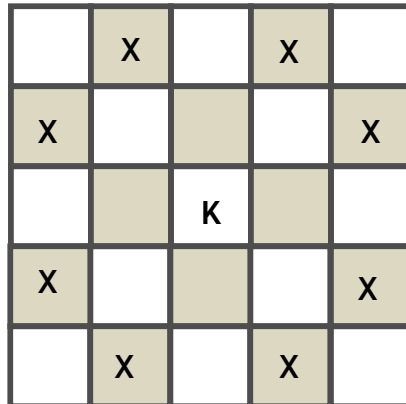
VII.A Student Activity Sheet 4: Hamiltonian Circuits and Paths

5. **REFLECTION:** Compare and contrast a Euler circuit and a Hamiltonian circuit.
6. **EXTENSION:** Describe a situation (other than travel) that requires a Hamiltonian circuit exist, but not a Euler circuit. Include either a diagram and graph or similar diagrams that show the connection of the graph to the real situation. Provide any details necessary to connect to the real-world application of this learning.

Networks and Graphs: Circuits, Path, and Graph Structures

VII.A Student Activity Sheet 5: Knight’s Tour

One game piece in chess is called a *knight*, and it is usually represented by a horse-like figure. This is the only chess piece that can jump over other pieces. An allowable move for a knight consists of moving two squares in one direction, followed by turning 90°, and then moving one square. A knight cannot move diagonally. The following figure shows a portion of a chessboard and all the possible squares (X) the knight (K) could go to in one move.



Knight’s Tour Problem

A traditional chessboard consists of an 8-by-8 grid with 64 squares. For the purposes of this activity, only a 4-by-4 grid is considered.

If a knight is placed in the upper left-hand square, is there a sequence of moves that allows the knight to visit every square on the chessboard exactly once and then return in the last move to its starting place? The remainder of this activity sheet will help you answer this question.

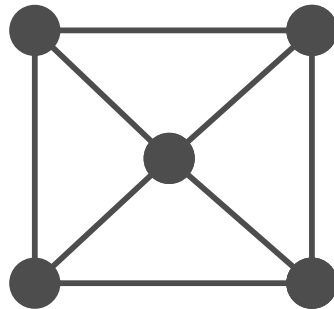
1. Decide how to use a graph to model the Knight’s Tour problem. Carefully define what your vertices represent and how you know when two vertices are connected by an edge.

Networks and Graphs: Spanning Trees

VII.B Student Activity Sheet 6: High-speed Internet

Your company must run Ethernet cables to five different offices so that all five offices have high-speed Internet access. For each computer to be on the office network, there must be a way to get from each computer to the other computers by following the cable.

1. One worker proposed running cable between the five offices as illustrated in the following diagram. The vertices represent the offices, and the edges represent segments of cable.



How many lengths of cable (edges) are used? Explain why this an inefficient way to run the cable.

2. Design a more efficient network and indicate how many lengths of cable are used.

Networks and Graphs: Spanning Trees

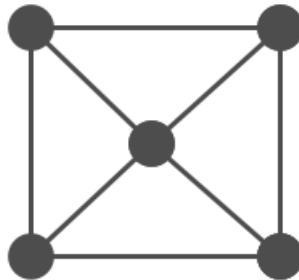
VII.B Student Activity Sheet 6: High-speed Internet

3. Compare your efficient network with others in the class.
- a. Did everyone use the same number of cable lengths?

b. Did everyone's network have the same shape?

4. A *cycle* in a graph is a path that starts and ends at the same vertex and does not use any edge more than once.

a. Identify two cycles in the graph from Question 1.



b. Does your network from Question 2 have any cycles? Should it?

c. What does the existence of cycles tell you about the efficiency of a network?

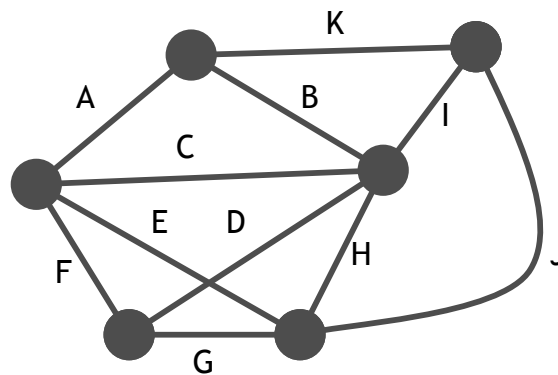
Networks and Graphs: Spanning Trees

VII.B Student Activity Sheet 6: High-speed Internet

5. **REFLECTION:** Describe how a cycle is similar to a Euler circuit.

6. Write a set of step-by-step instructions to form an algorithm for converting an inefficient network into an efficient network.

7. Have another student apply your algorithm to the following inefficient network and indicate the number of edges in the final efficient network.



Networks and Graphs: Spanning Trees

VII.B Student Activity Sheet 6: High-speed Internet

8. **EXTENSION:** Research other situations that might be modeled with graphs in such a way that cycles become important. Draw an efficient network for the situation (not a cable connection, of course!). Prepare a short presentation for the class.

Use the following questions as needed:

- Do all graphs have cycles?
- Are the cycles unique?

Networks and Graphs: Spanning Trees

VII.B Student Activity Sheet 7: Minimal Spanning Trees

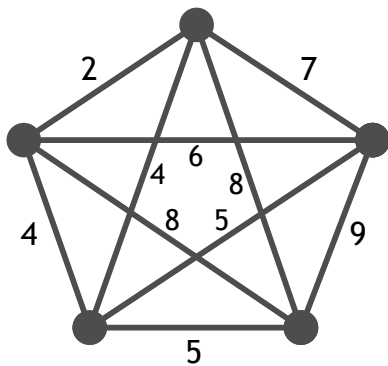
3. Construct two different graphs that represent possible railroad networks, and calculate the total cost to build each rail system.

4. What rail system leads to the lowest construction cost?

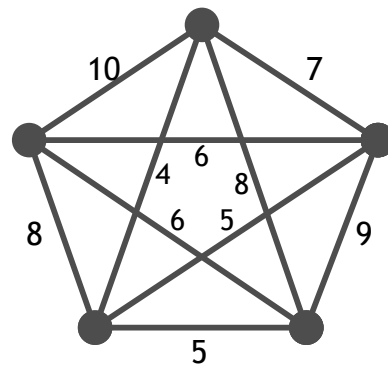
Networks and Graphs: Spanning Trees
 VII.B Student Activity Sheet 7: Minimal Spanning Trees

5. How did you arrive at the most efficient rail system? State your answer in a step-by-step algorithm that someone else could follow.

6. The numbers in the following graphs depict the cost associated with building a railroad between cities represented by vertices. For each graph, test your algorithm. Does it yield the most efficient network? If not, try modifying your algorithm.



Network I



Network II

Networks and Graphs: Spanning Trees

VII.B Student Activity Sheet 7: Minimal Spanning Trees

7. **REFLECTION:** Given any weighted graph (like the ones from this activity), does an efficient network of minimal cost always exist? Why or why not?
8. **EXTENSION:** What other real-world problems might be solved by creating and analyzing graphs with weighted edges? Conduct research to respond to this question, and prepare a short presentation for the class.

Networks and Graphs: Spanning Trees

VII.B Student Activity Sheet 8: Kruskal's Algorithm

A graph whose edges are given numerical values is called a *weighted graph*. Keeping all the vertices connected by a path resulting in a minimum total weight is called *finding a minimal spanning tree*. The word *spanning* means that each vertex remains connected to the graph, and the word *tree* indicates that there are no cycles.

The following procedure, known as Kruskal's Algorithm, can be used to find a minimal spanning tree in a weighted graph.

Kruskal's Algorithm

Assume that you start with a table of the weights associated with each edge (just like the Railroad problem in Student Activity Sheet 7).

Step 1: Put all of the weights in a list from smallest to largest.

Step 2: Find the smallest weight in the list and include the associated edge and two vertices, as long as that does not create a cycle.

Step 3: Remove this weight from the list.

Step 4: Repeat Steps 2 and 3 until all vertices are connected.

Networks and Graphs: Spanning Trees
 VII.B Student Activity Sheet 8: Kruskal’s Algorithm

1. A series of bridges will be constructed to connect a group of seven islands. The highway department wants to make sure that a vehicle can be driven from one island to the others in this new network of bridges. The cost of building a bridge is directly proportional to the length of the bridge. The following table provides the distances in miles between each pair of islands.

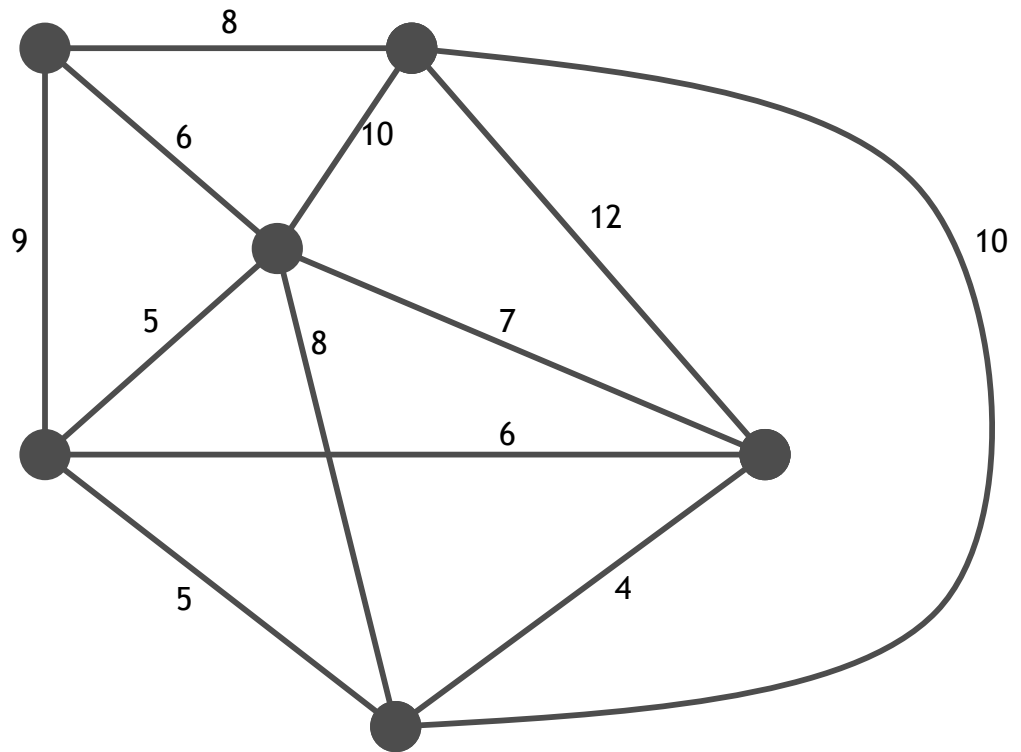
	Island A	Island B	Island C	Island D	Island E	Island F	Island G
Island A		10	8	8	7	10	9
Island B			4	9	13	3	7
Island C				12	11	5	9
Island D					9	10	6
Island E						6	11
Island F							8

Use Kruskal’s Algorithm to determine which islands should be connected by bridges. Draw a graph that represents the seven islands with the bridges that will be constructed.

Networks and Graphs: Spanning Trees

VII.B Student Activity Sheet 8: Kruskal's Algorithm

2. Use Kruskal's Algorithm to find a minimal spanning tree in the following graph.



Networks and Graphs: Spanning Trees

VII.B Student Activity Sheet 8: Kruskal's Algorithm

3. **REFLECTION:** Do all graphs have spanning trees? Are spanning trees unique?
4. **EXTENSION:** Other algorithms exist for arriving at a minimal spanning tree. Conduct research to find one and share it with your class through a brief presentation. Compare and contrast your findings with Kruskal's Algorithm as appropriate.

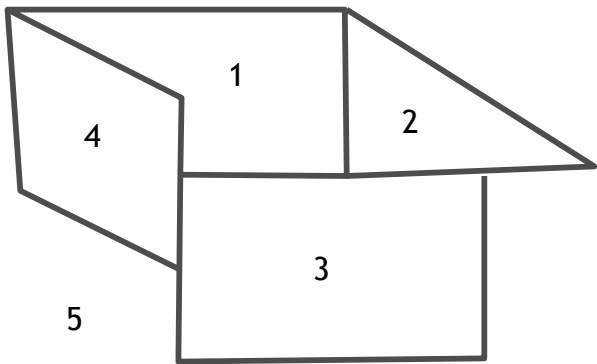
Networks and Graphs: Graph Coloring

VII.C Student Activity Sheet 9: Map Coloring

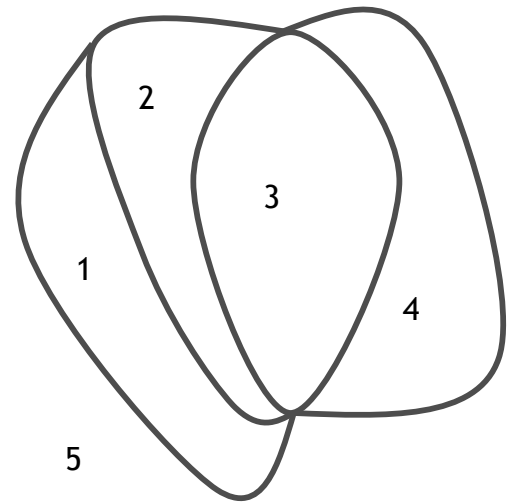
Map Coloring Problem

You are the publisher of a new edition of the world atlas. As you prepare the different maps for printing, you need to make sure that countries adjacent to each other (sharing a common border) are given different colors.

1. For the following two maps, decide how to color each of the five countries (regions) so that no two adjacent countries are colored the same. Treat the outside region as a single country (perhaps it represents an ocean colored blue). Assume that every country is composed of a single contiguous region (for example, you treat Alaska and Hawaii as separate regions when constructing a map of the world).



Map I



Map II

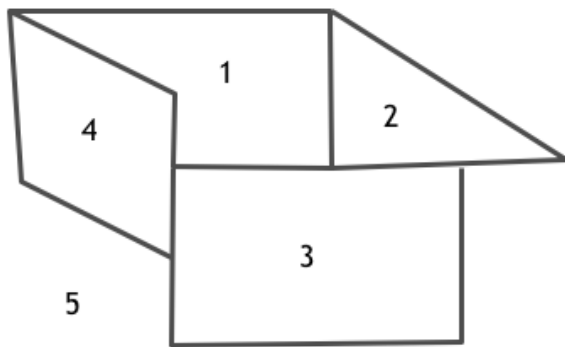
Networks and Graphs: Graph Coloring

VII.C Student Activity Sheet 10: Coloring Maps and Scheduling

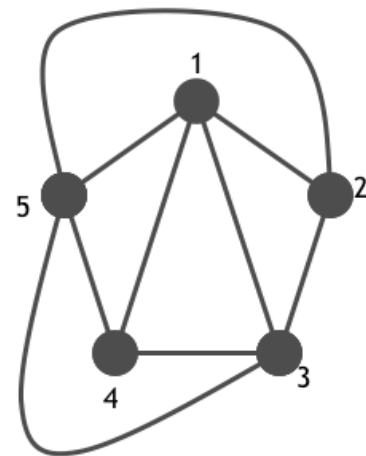
Creating Graphs from Maps

1. Revisit the map coloring exercises from Student Activity Sheet 9 in terms of graphs. For example, Map I can be represented by the following graph. The graph should include a vertex for each country (or region) in your map. If two countries share a border and need to be colored differently, the graph shows an edge between the vertices that represent them.

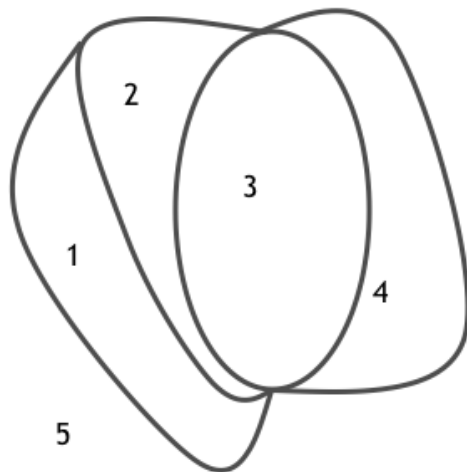
After studying the relationship between Map I and the graph for Map I, create a graph that represents Map II.



Map I



Graph for Map I



Map II

Networks and Graphs: Graph Coloring

VII.C Student Activity Sheet 10: Coloring Maps and Scheduling

2. Restate the Map Coloring problem from Student Activity Sheet 9 in terms of a Graph Coloring problem.

You are the publisher of a new edition of the world atlas. As you prepare the different maps for printing, you need to make sure that countries adjacent to each other (sharing a common border) are given different colors.

3. Create a graph that **requires** three colors.

4. Create a graph that **could be** colored with two colors.

Networks and Graphs: Graph Coloring

VII.C Student Activity Sheet 10: Coloring Maps and Scheduling

5. What types of graphs can always be colored with two colors?

6. **EXTENSION:** Create a graph that needs five colors, and then draw the associated map.

Networks and Graphs: Graph Coloring

VII.C Student Activity Sheet 10: Coloring Maps and Scheduling

7. **REFLECTION:** When might a graph not correspond to a map?

8. The *chromatic number* of a graph is the minimum number of colors needed to color each vertex in such a way that any two vertices sharing an edge are a different color. Provide examples of graphs that have chromatic numbers of 3 and 4.

Networks and Graphs: Graph Coloring

VII.C Student Activity Sheet 10: Coloring Maps and Scheduling

9. Give an example of a graph with 20 vertices that has a chromatic number of 2. Does your graph have any cycles? (*Recall:* A cycle is a path through the graph that starts and ends at the same vertex and does not reuse any edges.)

Networks and Graphs: Graph Coloring

VII.C Student Activity Sheet 10: Coloring Maps and Scheduling

Scheduling Problem

Mrs. Jacobs, the new principal at Riverdale High School, wants to make a good impression by offering a lot of new exciting classes for her students. The principal plans to use her knowledge of graph theory to determine when each class will be offered.

Since she is trying to make her students happy, Mrs. Jacobs does not want to offer two different classes at the same time if there are students wanting to take both. She decides to construct a graph in the following way: Each class is represented by a vertex and if there is a student interested in two classes, those two vertices are connected by an edge.

10. Suppose there are five classes (A, B, C, D, and E) and only five students wishing to take the following classes:

- Jason wants to take Classes A and E.
- Emory want to take Classes B, C, and E.
- Felicity wants to take Classes A and D.
- Geoff wants to take Classes B and C.
- Hilary wants to take Classes D and E.

Construct the graph for the principal.

Student: _____ Class: _____ Date: _____

Networks and Graphs: Graph Coloring

VII.C Student Activity Sheet 10: Coloring Maps and Scheduling

11. Find the chromatic number of the graph, and color the graph using the least number of colors.

12. How can the graph coloring solution help the principal with her scheduling problem?

Networks and Graphs: Graph Coloring

VII.C Student Activity Sheet 10: Coloring Maps and Scheduling

- 13. EXTENSION:** Select another situation that might be modeled with colored graphs. Several suggestions are described to stimulate your research. Prepare a short presentation of your findings to share with the class.

The notion of coloring graphs can be used to solve a variety of problems involving various types of conflicts over space or time. Some examples include the following:

- **Conflict over time:** Virtually any type of scheduling problem such as appointments or job duties based on an individual’s qualifications.
- **Conflict over space:** **1)** Create several terrariums to display a variety of plants and reptiles. Certain reptiles may not get along with others, and certain plants should not be placed in terrariums with certain reptiles. Based on a set of compatibility conditions, you could decide the minimum number of terrariums necessary for the exhibit. **2)** Radio stations that are within a certain distance of each other cannot be assigned the same broadcasting frequency. Given several radio stations and the distances between each pair, determine the minimum number of distinct frequencies necessary to allow all stations to operate.
- **Other conflicts:** Put a roomful of people into small working groups. Each individual may have a list of others with whom he/she does not work well, thereby disallowing them to share a group. Given each person’s “Cannot Work With” list, how many groups are necessary?
- **Chemistry:** Certain chemicals cannot be stored with other chemicals. For example, to answer the question regarding how many storage facilities are required to house the following chemicals, graph coloring can be helpful.

Chemicals	Cannot be stored with
1	2, 5, 7
2	1, 3, 5
3	2, 4
4	3, 7
5	1, 2, 6, 7
6	5
7	1, 4, 5

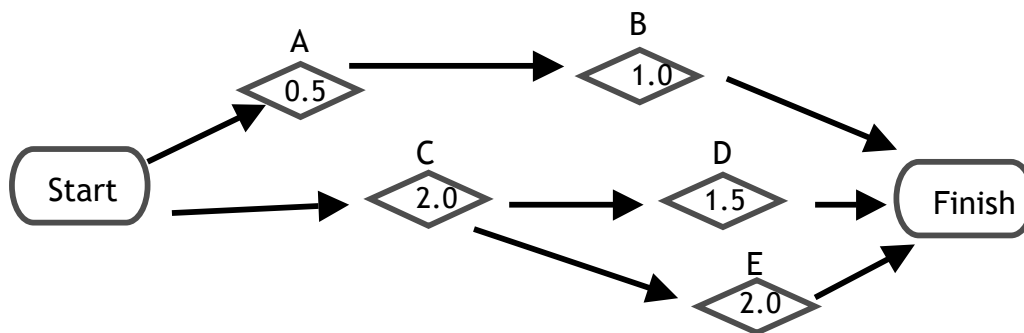
Networks and Graphs: Program Evaluation and Review Technique (PERT) Charts

VII.D Student Activity Sheet 11: Activity Graphs

You are in charge of organizing the senior class party. The following is your estimate of the time required to perform all the necessary activities:

Activity	Time (in hours)
A. Plan music playlist	0.5
B. Download music	1.0
C. Buy groceries and decorations	2.0
D. Bake cake and prepare food	1.5
E. Set up	2.0

Since there are several classmates helping, some of these tasks can be performed at the same time. For instance, people can begin setting up the decorations while the cake is baking. However, you cannot bake the cake or set up until after the shopping has taken place. And you cannot begin downloading music until you know what songs you want to download. The following *activity graph* can be used to help organize this information:



1. What do the numbers in this graph represent?
2. Why is there an arrow going from Activity A to Activity B, but not from Activity A to Activity C?

Networks and Graphs: Program Evaluation and Review Technique (PERT) Charts

VII.D Student Activity Sheet 11: Activity Graphs

Scheduling classes in college can be very similar to the previous scenario. Over four years, there are certain classes that you must take, and many classes have prerequisites—classes that must be taken first. Suppose you need to take the following classes with the identified prerequisites.

Class	Prerequisite
Calculus I	None
Calculus II	Calculus I
Physics I	Calculus I
Physics II	Physics I
Psychology I	None
Speech	None
Argument and Debate	Speech

7. Construct an activity graph for this situation using the following rules:

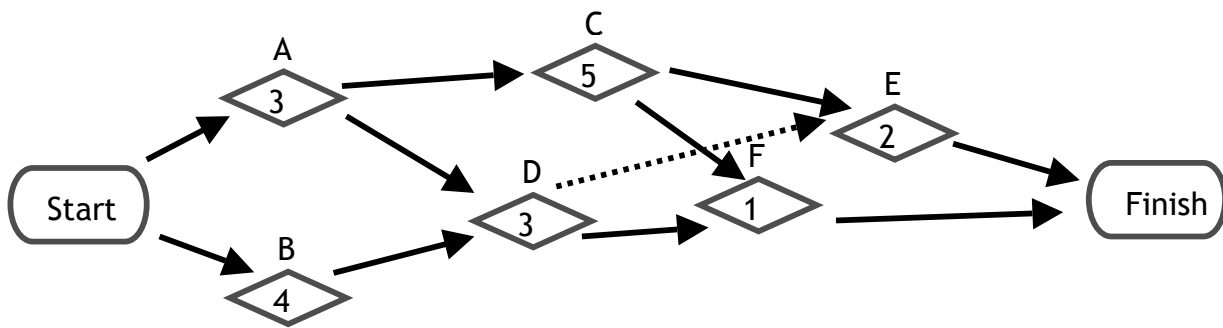
- Create Start and Finish squares.
- Any activity that can be performed right away is connected to Start.
- Activity A is connected to Activity B by an arrow **only when** Activity A needs to be performed directly before Activity B.
- Any activity that does not precede any other activity can be connected to Finish.

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VII.D Student Activity Sheet 11: Activity Graphs

13. Given any activity graph like the previous ones, explain how you would determine the minimum time required to perform all activities.
14. Activities that cannot be delayed without increasing the minimum time for completion are called *critical activities*. Given any activity graph like the previous ones, explain how you would determine which activities are critical activities.

For the following activity graph, the times given for each activity are in hours.



15. Determine the minimum time required to complete all the activities shown in the graph.

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VII.D Student Activity Sheet 11: Activity Graphs

20. **REFLECTION:** Can you find a formula that determines how long an individual activity could be delayed without affecting the total completion time for all the activities?

Sometimes the time to complete an activity is given by two numbers: the estimate for a minimum completion time and the estimate for a maximum completion time. How would having two possible completion times affect your analysis?

21. **EXTENSION:** Design a chart to represent the planning and design of a particular event at your school, (for example, Project Graduation, prom, fundraiser, community project). Prepare a short presentation including appropriate visuals to share with the class.

Networks and Graphs: Program Evaluation and Review Technique (PERT) Charts

VII.D Student Activity Sheet 12: Building a Robot

You are leading a group that is designing and building a robot; the group is divided into several teams. The following table indicates the different activities that go into this complex process, which teams are in charge of which activities, the number of individuals from that team dedicated to that activity, how long the activity will likely take, and which activities must be completed before an activity can be started.

Activity	Time to Complete (in Weeks)	Must First Finish Activity...	Team	No. of Members
Sensory Program (SP)	4	None	Computer programmers	3
Artificial Intelligence (AI)	5	None	Computer programmers	3
Motion (M)	10	SP, AI	Computer programmers	3
Voice System (VS)	3	AI	Computer programmers	3
Eye Design (ED)	5	SP	Engineers	3
Arm and Leg Design (ALD)	3	M	Engineers	3
Body Design (BD)	4	None	Engineers	3
Head Assembly (HA)	5	VS, ED	Technicians	3
Torso Assembly (TA)	2	BD	Technicians	3
Arm and Leg Assembly (ALA)	8	ALD	Technicians	3
Appearance (A)	2	TA, ALA	Technicians	3
Final Assembly (FA)	4	HA, TA, ALA	Technicians	3

Networks and Graphs: Program Evaluation and Review Technique (PERT) Charts

VII.D Student Activity Sheet 12: Building a Robot

5. **EXTENSION:** Since any delay in the completion time for critical activities results in a longer total completion time, these activities may need extra people assigned to them. Suppose you can reassign team members to an activity according to the following guidelines:

- No one can work on an activity outside of his/her team. For example, a computer programmer must be assigned to Activity 1, 2, 3, or 4 and cannot be assigned to any of the other activities.
 - Every activity must have at least one person assigned to it at all times.
 - An activity that receives extra help can be completed 1 week earlier for each additional person assigned to it.
 - An activity cannot be completed in less than 1 week, even if more people are assigned to it.
 - An activity takes 1 week longer to complete for each person removed from the original group.
- a. If you could reassign one person, how would you do it? How does the reassignment affect the total completion time?
- b. If you could reassign two people, how would you do it? How do the reassignments affect the total completion time?

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VII.D Student Activity Sheet 12: Building a Robot

- c. If you could reassign any number of people, how would you do it? How do the reassignments affect the total completion time?

6. **REFLECTION:** Could the total completion time be further improved by allowing people to work on activities outside of their official team designation? Justify your response with appropriate reasoning.